

Find the point-slope form of the equation of the **normal** line to the curve $y = \frac{x^2}{f(x)}$ at the point where $x = -3$ SCORE: ____ / 5 PTS

if $f(-3) = -2$ and $f'(-3) = 5$.

$$\frac{dy}{dx} = \frac{2xf(x) - x^2f'(x)}{[f(x)]^2} \quad (2)$$

$$x = -3 \rightarrow y = \frac{(-3)^2}{f(-3)} = -\frac{9}{2}$$

$$\left. \frac{dy}{dx} \right|_{x=-3} = \frac{2(-3)f(-3) - (-3)^2f'(-3)}{[f(-3)]^2} = \frac{12 - 45}{4} = -\frac{33}{4} \quad (1)$$

$$\left(\frac{1}{2} \right) y + \frac{9}{2} = \frac{4}{33} (x + 3) \quad \left(\frac{1}{2} \right)$$

Prove the derivative of $\csc x$ using the quotient rule. **Show all steps.**

SCORE: ____ / 3 PTS

$$\frac{d}{dx} \frac{1}{\sin x} = \frac{0(\sin x) - 1(\cos x)}{\sin^2 x} \quad \left(\frac{1}{2} \right)$$

$$= -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} \quad (1)$$

$$= -\cot x \csc x \quad \left(\frac{1}{2} \right)$$

Prove the derivative of $\tan x$ using the definition of the derivative function. **Show all steps.**

SCORE: ____ / 5 PTS

GRADED BY ME

The position of an object at time t is given by $s(t) = e^t \sin t$.

SCORE: ____ / 4 PTS

Find the acceleration of the object at time $t = \frac{\pi}{2}$.

$$s'(t) = e^t \sin t + e^t \cos t \quad (1)$$

$$s''(t) = (e^t \sin t + e^t \cos t) + (e^t \cos t - e^t \sin t) \quad (2)$$

$$= 2e^t \cos t \quad (1/2)$$

$$s''(\frac{\pi}{2}) = 2e^{\frac{\pi}{2}}(0) = 0 \quad (1/2)$$

Find the following derivatives. Simplify all answers appropriately.

SCORE: ____ / 13 PTS

[a] $\frac{d^3}{dx^3} \frac{9x^2 - 18x}{\sqrt[3]{x}}$

$$= \frac{d^3}{dx^3} (9x^{\frac{5}{3}} - 18x^{\frac{2}{3}}) \quad (1/2)$$

$$= \frac{d^2}{dx^2} (15x^{\frac{2}{3}} - 12x^{-\frac{1}{3}}) \quad (1)$$

$$= \frac{d}{dx} (10x^{-\frac{1}{3}} + 4x^{-\frac{4}{3}}) \quad (1)$$

$$= -\frac{10}{3}x^{-\frac{4}{3}} - \frac{16}{3}x^{-\frac{7}{3}} \quad (1)$$

[b] $\frac{d}{db} \frac{1-5b^3}{4b-b^2} \quad (1)$ (Your final answer must be a single fraction)

$$= \frac{-15b^2(4b-b^2) - (1-5b^3)(4-2b)}{(4b-b^2)^2}$$

$$= \frac{-60b^3 + 15b^4 - 4 + 2b + 20b^3 - 10b^4}{(4b-b^2)^2}$$

$$= \frac{5b^4 - 40b^3 + 2b - 4}{(4b-b^2)^2} \quad (1)$$

[c] $\frac{d}{dx} (3t^x + \frac{1}{e^t} - 5x^e)$

$$= \underbrace{3t^x \ln t}_{(1)} - \underbrace{5ex^{e-1}}_{(1)}$$

MINUS (1) POINT

IF YOU HAD ANY OTHER
TERMS IN THIS DERIVATIVE

eg. $-e^{-t}$

[d] $\frac{d}{dx} (\cot x \cos x - \sec x) \quad (1)$

$$= \underbrace{-\csc^2 x \cos x}_{(1)} - \underbrace{\cot x \sin x}_{(1)} - \underbrace{\sec x \tan x}_{(1)}$$

$$= -\csc x \cot x - \underbrace{\cos x}_{(1/2)} - \sec x \tan x$$